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Primordial magnetic fields from inflation???[1]

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Abstract

In this note we argue that the breaking of conformal invariance because of the coupling of a charged scalar field to gravity is not sufficient for the production of seed galactic magnetic fields during inflation.

Since the problem of long ranged magnetic fields is well known and is thoroughly discussed in the literature (for a very recent review see [1]) we are not going to describe it here. We will instead concentrate on the recent controversy concerning the question of production of primordial seed magnetic fields from scalar field fluctuations during inflation.

The original idea, due to Turner and Widrow [2], can be formulated as follows. While the coupling of electromagnetic field to the metric and to the charged fields is conformally invariant, the coupling of the charged scalar field to gravity is not. Thus, vacuum fluctuations of the charged scalar field can be amplified during inflation over super-horizon scales, leading, potentially to non-trivial correlations of the electric currents and charges over cosmologically interesting distances. The fluctuations of electric currents, in their turn, may induce magnetic fields through Maxwell equations at the corresponding scales. The role of the charged scalar field may be played by the Higgs boson which couples to the hypercharge field above the electroweak phase transition. The generated hypercharged field is converted into ordinary magnetic field at the temperatures of the order of electroweak scale.

This suggestion was further investigated in [3] for the standard electroweak theory with a conclusion that large scale magnetic fields can be indeed generated. These estimates were challenged in [4], where the computation similar to [3] has been carried out, but with the adequate treatment of kinetics of the hot plasma and realistic dependence of electric conductivity on

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the temperature. Moreover, in [4] the accurate expression for the amplified currents has been derived without the use of WKB approximation (which has been assumed in [3]). The inclusion of these effects changed the estimates of [3] to a level not sufficient for seeding of the galactic magnetic fields.

Very recently, a new proposal, also based on inflation, was put forward [5] (see also [6]). The authors use the fact that during the exponential expansion of the Universe the long ranged fluctuations of the charged scalar field are amplified. This leads to the mass generation for the vector field and, therefore, to the breaking of U(1) symmetry. The inflationary phase is then replaced by a radiation dominated stage of expansion, and the scalar field fluctuations relax to zero, leading to the restoration of the U(1) symmetry. In Ref.[5] it was claimed that the abrupt change of the mass of the photon at the end of the inflationary stage results in production of magnetic fields which may be sufficiently large to seed the galactic magnetic fields, and in ref. [6] it was claimed that the coherent oscillations of the charged scalar field during preheating lead to the similar effect.

In the conformal time coordinate τ, the authors of [5, 6] write the evolution equation of the fluctuations of the gauge fields as:

$$\left(\partial_\tau^2 + \vec{k}^2 + e^2 a^2 \langle\rho^2\rangle\right) A_i(\vec{k},\tau) = 0 \ , \tag{1}$$

where e is the charge of scalars, a is the scale factor, $\langle\rho^2\rangle$ is the magnitude of the scalar field fluctuations (computed in the unitary gauge), and $A_i(\vec{k},\tau)$ is the Fourier harmonic of the vector field. Taking a step-function approximation for the change of the scalar field fluctuations the authors of [5] found that the resulting spectrum of the generated field strengths is approximately $B_l \propto l^{-1}$, where l is the relevant coherence scale. In ref.[6] an oscillating behaviour of the scalar field was assumed.

Our main objection to the analysis of refs. [5, 6] is that Eq. (1) disregards dissipative effects that are crucially important for the generation of magnetic fields and charge density fluctuations. Though the importance of dissipation was mentioned in refs. [5, 6], it was not taken into account in computation of amplification of gauge field fluctuations. While in de Sitter stage of expansion one can assume that the equations of motion in the vacuum are adequate, they are certainly not true in radiation dominated epoch containing a plasma of charged particles. In particular, these equations are not correct during the preheating stage, where charged scalars are copiously produced simply because of their self-interaction. They are also not true right after inflation, since the change of the gravitational background leads to creation of charged scalar particles. In the media, containing charged particles, the change of vector potential gives rise to electric fields that accelerate particles, leading, ultimately, to damping effects. A phenomenological way[4] to incorporate the damping effects is to add to the left-hand side of Eq. (1) a damping term [3]

$$\sigma a \partial_\tau A_i(\vec{k},\tau) \ , \tag{2}$$

[4] A more rigorous treatment, based on the Landau-Vlasov kinetic equation, is discussed in [4].

where σ is a plasma conductivity. The lower limit on the conductivity can be established simply by the counting of the number of scalar particles produced during inflation and is given by equations (5.8) and (B.23) of Ref. [4], which lead to $\sigma \sim \frac{H}{\alpha}$, where H is the Hubble constant during inflation and α is the fine structure constant. One can check that that the damping term is greater than the one with the second derivative, so that the damping (rather than amplification!) of the gauge field fluctuations is given by

$$A_i(\vec{k},\tau) = \exp\left(-\int_{\tau_0}^{\tau} d\tau \frac{e^2 a^2 \langle \rho^2 \rangle}{\sigma a}\right) A_i(\vec{k},\tau_0)\ , \tag{3}$$

rather than what was found in Refs.[5, 6]. Here $A_i(\vec{k},\tau_0)$ is the typical gauge field fluctuation at the end of inflation.

The effect of the conductivity is crucial exactly because the fluctuations of the massive vector field *do not grow* during inflation and in fact they are smaller than the ones of a free massless vector field that couples to gravity in a conformally-invariant way. This follows from the analysis of the correlation functions of the magnetic fields during the de Sitter phase, based on eq. (1),

$$\langle B_i(\vec{x},\tau) B_j(\vec{y},\tau)\rangle = \int d^3k\ \tau\ P_{ij}(k) H_\mu^{(1)}(k\tau)\ H_\mu^{(2)}(k\tau) e^{i\vec{k}\cdot(\vec{x}-\vec{y})}, \tag{4}$$

where

$$P_{ij}(k) = \frac{k^2}{32\ \pi^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right), \tag{5}$$

and $H_\mu^{(1)}(k\tau) = H_\mu^{(2)^*}(k\tau)$ are the Hankel functions of first and second order. As a consequence the spectrum of magnetic fields (in terms of the length scale l) is of the form $B_l \propto l^{-\nu}$ where $\nu = 2 + \mathcal{O}(\frac{e^2}{\lambda}) > 2$. Thus, the amplitude of the magnetic field, created by this mechanism, is too small on cosmological distances.

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